

Physics 128 Discussion (Week 2)

Problem 1

(a) The *radiated energy flux* of the Sun on Earth is approximately $1400 \text{ W m}^{-2} = 1400 \text{ J m}^{-2} \text{ sec}^{-1}$. The Earth's radius is about $6 \times 10^6 \text{ m}$, and the Earth is about 10^{11} m from the Sun. What is the total approximate power emitted by the Sun (the Sun's *intrinsic luminosity*)? If Earth absorbed all the solar energy it received, how much solar energy would it absorb in one minute? Assume the Sun is an isotropic point source.

Let S be the area of the sphere with radius $1 \text{ AU} = 10^{11} \text{ m}$, and $F_{\text{Earth}} = 1400 \text{ J m}^{-2} \text{ sec}^{-1}$ be the energy flux on earth, then the total luminosity $L_{\odot}$ satisfies

$$\begin{aligned} F_{\text{Earth}} &= \frac{L_{\odot}}{S} \\ &= \frac{L_{\odot}}{4\pi (1 \text{ AU})^2} \quad (1 \text{ AU} = 10^{11} \text{ m : astronimical unit}) \end{aligned} \quad (1)$$

$$\begin{aligned} \Rightarrow L_{\odot} &= 4\pi (1 \text{ AU})^2 F_{\text{Earth}} \\ &= 4\pi (10^{11} \text{ m})^2 (1400 \text{ J m}^{-2} \text{ sec}^{-1}) \\ &= 12.56 \times 10^{22} \text{ m}^2 \times (1.4 \times 10^3 \text{ J m}^{-2} \text{ sec}^{-1}) \\ &\simeq 1.76 \times 10^{26} \text{ J sec}^{-1} \end{aligned} \quad (2)$$

Earth absorbs the solar radiation on hemisphere then the total energy received in one minute is given by

$$\begin{aligned} E_{\text{recieved}} &= 2\pi (6 \times 10^6 \text{ m})^2 F_{\text{Earth}} \times (60 \text{ s}) \\ &\simeq 1.90 \times 10^{19} \text{ J} \end{aligned} \quad (3)$$

(b) Assuming that the solar luminosity is totally coming from the energy release of the pp -chain reaction:



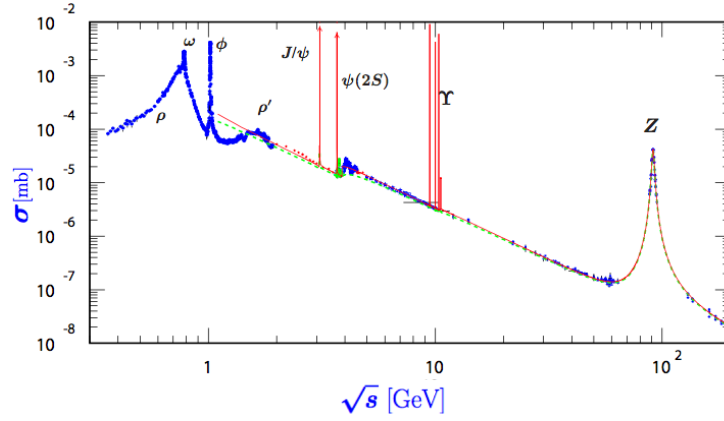
calculate the rate of this reaction in the sun per second.

The total luminosity $L_{\odot}$ divided by $\Delta E = 26.73 \text{ MeV}$ gives you the pp -chain reaction rate R inside the sun

$$\begin{aligned} R &= \frac{L_{\odot}}{\Delta E} \\ &\simeq \frac{1.76 \times 10^{26} \text{ J sec}^{-1}}{26.73 \text{ MeV}} \\ &\simeq 4.11 \times 10^{37} \text{ sec}^{-1} \end{aligned} \quad (5)$$

Problem 2

Here is some actual data of total cross section measurement of hadron creations from e^+e^- annihilation experiments



(a) From this plot, read out the decay width of ω, ρ' mesons, and Z boson.

$$\Gamma_\omega \approx 0.01 \text{ GeV} \quad (6)$$

$$\Gamma_{\rho'} \approx 0.1 \text{ GeV} \quad (7)$$

$$\Gamma_Z \approx 1 \text{ GeV} \quad (8)$$

(b) Estimate the lifetime of ω, ρ' mesons, and Z boson.

$$\tau_\omega = \frac{\hbar}{\Gamma_\omega} \approx 6.58 \times 10^{-23} \text{ s} \quad (9)$$

$$\tau_{\rho'} = \frac{\hbar}{\Gamma_{\rho'}} \approx 6.58 \times 10^{-24} \text{ s} \quad (10)$$

$$\tau_Z = \frac{\hbar}{\Gamma_Z} \approx 6.58 \times 10^{-25} \text{ s}. \quad (11)$$

Problem 3

In a collision experiment, 10^6 protons are incident on a target with a thickness of 0.01 cm and an atomic number density of $6.0 \times 10^{22} \text{ cm}^{-3}$.

(a) When the proton energy is 100 GeV, 100 collision events are observed. For a proton energy of 1 TeV, the number of observed collision events increases to 1000. Calculate the total cross section for each energy.

The probability of collision P is given by

$$P \equiv \frac{N_{\text{collision}}}{N_{\text{initial}}}. \quad (12)$$

You can also express the same probability by using the mean free path $\lambda \equiv (n_{\text{target}}\sigma)^{-1}$

$$\begin{aligned} P &= 1 - e^{-\frac{d}{\lambda}} \quad \left(e^{-\frac{d}{\lambda}} : \text{survival probability} \right) \\ &= 1 - e^{-n_{\text{target}}\sigma d} \end{aligned} \quad (13)$$

where $d = 0.01 \text{ cm}$ is the thickness of the target, and $n_{\text{target}} = 6.0 \times 10^{22} \text{ cm}^{-3}$ is the atomic number density of the target. Rearranging for σ , you get

$$\sigma = -\frac{\log(1 - P)}{n_{\text{target}}d}. \quad (14)$$

For $E = 100 \text{ GeV}$,

$$\begin{aligned} P &= \frac{100}{10^6} = 10^{-4} \\ \sigma &\approx 1.67 \times 10^{-25} \text{ cm}^2 \\ &= 0.167 \text{ b} \end{aligned} \quad (15)$$

For $E = 1 \text{ TeV}$,

$$\begin{aligned} P &= \frac{1000}{10^6} = 10^{-3} \\ \sigma &\approx 1.67 \times 10^{-24} \text{ cm}^2 \\ &= 1.67 \text{ b} \end{aligned} \quad (16)$$

(b) Discuss the energy dependence based on your results.

The results suggest an increase in the total cross section with higher proton energy, indicating a potential energy dependence in the interaction probability.¹

¹Some process shows the opposite feature, namely the total cross section decreases as the particle goes higher energies. A typical example is the Klein-Nishina cross section for the Compton scattering.

Problem 4

An extensive cosmic ray shower is the result of a collision of a very high energy proton from space with the nucleus of a molecule of air in the upper atmosphere. Consider for definiteness a collision that takes place at a height of 5 km. Take air (composed mainly of ^{14}N) to have a uniform density of 10^{-3} g/cm^3 , and rock (composed mainly of ^{28}Si) to have a uniform density of 2.6 g/cm^3 . About 1000 charged pions might be produced in a very high energy cosmic ray interaction.

(a) The probability of a π^+p interaction can be estimated by assigning an effective cross section of p

$$\sigma(A=1) = 3 \text{ fm}^2 = 3 \times 10^{-26} \text{ cm}^2. \quad (17)$$

Using this quantity, estimate the mean free path λ_i for this standard air and rock. (For $\pi^\pm$ -nucleus scattering, the cross section should be multiplied by $A^{2/3}$, where A is the nucleon number since the effective radius of the nuclei is proportional to $A^{1/3}$)

$$\sigma(A) = 3A^{2/3} \text{ fm}^2 = 3 \times 10^{-26} A^{2/3} \text{ cm}^2. \quad (18)$$

To solve this problem, we will calculate the mean free path λ_i of a charged pion in air and rock using the provided effective cross section formula and mass densities ρ of the material.

$$\lambda_i = \frac{1}{n\sigma(A)} \quad (\rho \equiv mn), \quad (19)$$

where n is the number density of the material m is the mass of one nucleus, given by $m = Am_u$ with atomic mass unit $m_u = \frac{1}{N_A} \text{ g} = 1.66 \times 10^{-24} \text{ g}$. That is

$$n = \frac{\rho}{m} = \frac{\rho}{A} N_A, \quad (20)$$

where N_A is Avogadro's number.

(I) For air (composed mainly of ^{14}N)

$$\begin{aligned} \rho &= 10^{-3} \text{ g/cm}^3, \quad A = 14 \\ \Rightarrow n_{\text{air}} &= 4.3 \times 10^{19} \text{ cm}^{-3}, \quad \sigma(A=14) = 1.74 \times 10^{-25} \text{ cm}^2 \end{aligned} \quad (21)$$

Then the mean free path in the air is given by

$$\begin{aligned} \lambda_{\text{air}} &= \frac{1}{n_{\text{air}}\sigma(A=14)} \\ &= \frac{1}{(4.3 \times 10^{19} \text{ cm}^{-3})(1.74 \times 10^{-25} \text{ cm}^2)} \\ &\approx 1.3 \times 10^5 \text{ cm} \\ &= 1.3 \text{ km} \end{aligned} \quad (22)$$

(II) For rock (composed mainly of ^{28}Si)

$$\begin{aligned} \rho &= 2.6 \text{ g/cm}^3, \quad A = 28 \\ \Rightarrow n_{\text{rock}} &= 5.6 \times 10^{22} \text{ cm}^{-3}, \quad \sigma(A=28) = 2.77 \times 10^{-25} \text{ cm}^2 \end{aligned} \quad (23)$$

Then the mean free path in the rock is given by

$$\begin{aligned} \lambda_{\text{rock}} &= \frac{1}{n_{\text{rock}}\sigma(A=28)} \\ &= \frac{1}{(5.6 \times 10^{22} \text{ cm}^{-3})(2.77 \times 10^{-25} \text{ cm}^2)} \\ &\approx 64 \text{ cm} \end{aligned} \quad (24)$$

(b) If a π^+ of 1 GeV is produced in the original p -nucleus collision, what is the probability that it suffers a nuclear collision before hitting the earth? What is the probability that it decays? The pion lifetime at rest is 2.6×10^{-8} s. The primary decay mode of the π^+ is

$$\pi^+ \rightarrow \mu^+ \nu, \quad (25)$$

with the μ^+ taking most of its energy.

From (a), we know that the mean free path of a pion collision in the air is

$$\lambda_{\text{air}} \approx 1.3 \text{ km}. \quad (26)$$

Then the probability of a pion undergoing a nuclear collision before reaching the Earth's surface is

$$\begin{aligned} P_{\text{collision}} &= 1 - e^{-\frac{h}{\lambda_{\text{air}}}} \\ &= 1 - e^{-\frac{5 \text{ km}}{1.3 \text{ km}}} \\ &\approx 0.98. \end{aligned} \quad (27)$$

The pion's lifetime in the lab frame is extended by the Lorentz factor

$$\begin{aligned} \gamma &= \frac{E}{m_{\pi} c^2} \\ &= \frac{1 \text{ GeV}}{0.14 \text{ GeV}} \\ &\approx 7.14, \end{aligned} \quad (28)$$

and this γ implies that the pion travels approximately the speed of light c . Thus, the lab frame lifetime is given by

$$\begin{aligned} \tau_{\text{lab}} &= \gamma \tau_{\pi} \\ &= 7.14 \times (2.6 \times 10^{-8} \text{ s}) \\ &\approx 1.86 \times 10^{-7} \text{ s}. \end{aligned} \quad (29)$$

Then, the pion travels a distance in the lab frame before decaying

$$\begin{aligned} d_{\text{decay}} &= c \tau_{\text{lab}} \\ &= 55.8 \text{ m}. \end{aligned} \quad (30)$$

Therefore the probability of a pion decaying before reaching the Earth's surface is

$$\begin{aligned} P_{\text{collision}} &= 1 - e^{-\frac{h}{d_{\text{decay}}}} \\ &= 1 - e^{-\frac{5 \text{ km}}{55.8 \text{ m}}} \\ &\approx 1.00. \end{aligned} \quad (31)$$

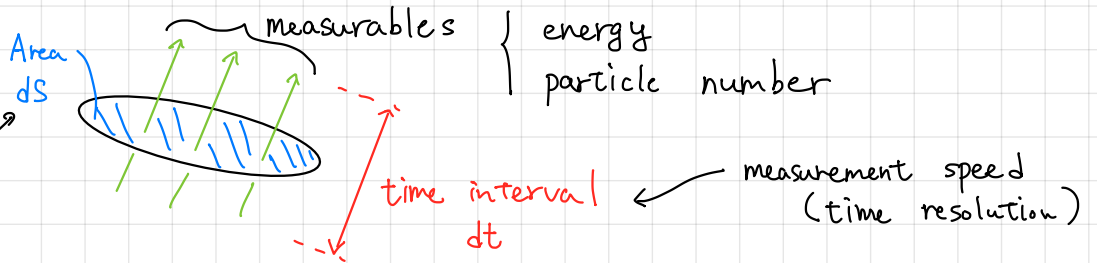
Thus, all the pions will decay to muons before reaching the Earth's surface.²

²These muons are a major source of experimental noise in the cosmic ray experiment. Some neutrino experiments are set in the old mines in the rocky mountains to screen the energetic muons, e.g., the Super Kamiokande experiment in Japan.

③ Flux :

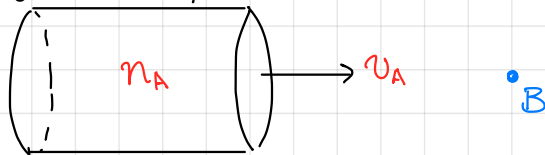
naive image

detector cell
(spatial resolution)



	energy flux	number flux	Luminosity (Power)
In SI unit	$\frac{\text{J}}{\text{m}^2 \cdot \text{sec}}$	$\frac{\#}{\text{m}^2 \cdot \text{sec}}$	$\frac{\text{J}}{\text{sec}} = \text{W}$
In natural unit	GeV^4	GeV^3	GeV^2

e.g. Fixed target experiment.



Event rate

$$\frac{\#}{(\text{time})} \propto (\text{number flux of A}) = n_A v_A$$

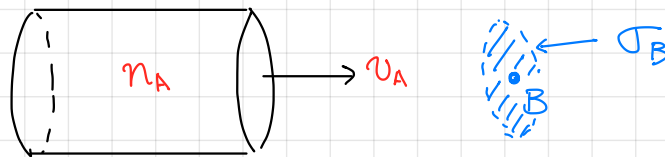
$$\Rightarrow \frac{\#}{(\text{time})} = n_A v_A \sigma_B$$

Dimensional analysis

$$[T]^{-1} = [L]^{-3} [L] [T]^{-1} [\sigma_B]$$

$$\Rightarrow [\sigma_B] = [L]^2$$

$\Rightarrow \sigma_B$ has a unit of "area" : "total cross section"



$$1 \text{ barn} = 10^{-28} \text{ m}^2 = 10^{-24} \text{ cm}^2$$

③ Convenient quantities

- mean-free-path of A : $\lambda_A = \frac{1}{n_B \sigma_B} : [L]^3 \cdot [L]^{-2} = [L]$
- Interaction timescale (lifetime) of A : $\tau_A = \frac{\lambda_A}{v_A} = \frac{1}{n_B \sigma_B v_A} : [T]$
- Interaction rate : $R_A = \frac{1}{\tau_A} : [T]^{-1}$
- Width : $\Gamma_A = \hbar R = \frac{\hbar}{\tau_A} : [\text{Energy}] \quad (\odot [\hbar] = [\text{Energy}] \cdot [T])$

e.g. Initial : No particles

particles number change within Δt : $\Delta N = -R N(t) \Delta t$

$$\leadsto \frac{dN}{dt} = -R N \Rightarrow N(t) = N_0 e^{-Rt}$$